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Reading Mathematical Texts: Cognitive Processes and Mental Representations

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There is a need to distinguish between understanding a text and learning from a text, something that can be done by looking at different levels of mental representations created when reading. This study is focusing on the mental representations created in two types of situations, where different kinds of cognitive processes are dominant, either a more perception-like process or a more problem-solving process. The purpose is to test an analytical tool of different levels of mental representations when used for mathematical texts, and to see examples of students' mental representations when reading different kinds of mathematical texts.

Introduction

This study focuses on reading mathematical texts, especially in relation to learning mathematics. Previous studies have often reduced reading within mathematics to an obstacle for learning (Borasi & Siegel, 1990, 1994), for example by focusing on comprehending word problems or on how deficits in reading ability affect the learning of mathematics. Thus, reading is seen as a necessary kind of knowledge, which only seems to interfere with learning. But reading could also be seen as a learning situation in itself, a view that serves as a foundation in this study. Thus, the focus will not be on reading in relation to the process of solving tasks or problems, but on understanding and learning when reading mathematical texts.

When focusing on the process of reading, the readability of mathematical texts is often discussed, where research sometimes uses readability formulas (e.g., Newton & Merrell, 1994) and sometimes gives suggestions on how to make texts easier to read (e.g., Hubbard, 1992). By doing this, without also discussing the relation between reading and learning, it is automatically assumed that easier texts in general are better for readers. But other studies, using texts from other subject areas, have noted a need to distinguish between different aspects of reading comprehension, especially readers' understanding and memory of the content of the text on one hand and learning from the text on the other. Some results show that readers with better prior knowledge learned less from a text that was easier to read (McNamara, 2001; McNamara, Kintsch, Songer, & Kintsch, 1996). These studies are founded in a theory of reading comprehension developed within cognitive psychology in the last 25 years by Kintsch (1998).

It thus seems necessary to differentiate between understanding the content of a text and learning from the text, a distinction that is included in the theory of Kintsch (1998). Parts of his theory will be used as a tool in the analysis in this study.

Reading

The theoretical foundation and the tools for analysis in this study are mostly from research of reading that does not primarily focus on mathematics, and much from the research by Kintsch (1998), where the only use of mathematics has been about word problems. There are of course differences between texts from different subject areas, and therefore it seems reasonable that there also are differences in *reading* texts from different subject areas. Brunner (1976) points to some

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special properties of mathematical texts, and states that the "differences stem from both the nature of the subject and the way it is written" (p. 208). But at the same time it seems natural that there exists a common core in all types of reading processes, independently of the content of the text, and also certain aspects of comprehension that can be relevant in all types of reading, or even more generally, in all types of situations. Kintsch (1998) describes these types of general aspects of comprehension, and his theory is not limited to reading but

> it is a psychological process theory. That is, it is concerned with the mental processes involved in acts of comprehension - not primarily with the analysis of the material that is to be comprehended. Applied to text comprehension, this means that it is not a theory of text structure, or a text analysis. The text structure is only indirectly important, in that it is one determinant of the comprehension process and therefore of the product of this process: the mental representation of the text and actions based on this construction (pp. 3-4).

Cognitive Processes

When reading, different kinds of cognitive processes come into play, and for the reader, these processes can be more or less conscious. Note that this is a distinction about the *progresses*, while the *results* of these processes are always considered conscious. Perception refers to highly automatic and unconscious processes, for example when you see a dog and directly recognize it as a dog; you are aware of the result of the process (that you see *a dog*) but no active and conscious thought processes were needed for this recognition. Problem solving on the other hand deals with active thinking, a more resource demanding process, for example when trying to remember the name of a person you meet and recognize.

In a given situation, some sort of mixture of these two types of processes usually exists. When reading a text without any difficulties in understanding what you read, the process has more in common with perception than with problem solving, in that the process of understanding is unconscious. This is a situation representative for Kintsch's (1998) concept of *comprehension*, which "is located somewhere along that continuum between perception and problem solving" (Kintsch, 1992, p. 144). But when a reader does not understand something in a text, problem-solving processes are needed to repair this, but may not always be activated by the reader. Reading thus has a comprehension aspect, which deals with automatic, unconscious cognitive processes; and a problem-solving aspect, which deals with active, conscious processes.

Mental Representations

Independent of the different aspects of cognitive processes when reading, the result of the reading process is a mental representation of the text:

> The resulting representation consists of nodes, which capture the elements in or related to the text, and connections, which capture the semantic relations between text elements. Together, these nodes and connections form a network.
>
> (van den Broek, Virtue, Everson, Tzeng, & Sung, 2002, p. 132).

However, the above mentioned connections need not be limited to semantic relations, but can in principle be based on just about anything, from linguistic, semantic and causal relations to associations based on personal experience.

Note that the mental representation resulting from reading a text does not only include text elements but also elements *related* to the text, that is, connections are created between text elements and existing elements in long-term memory, which is "everything a person knows and remembers" (Kintsch, 1998, p. 217). And this is in principle how a reader learns something from a text, by change of long-term memory. To refine this description one can distinguish between three different components, or levels, of the mental representation of texts; surface component, textbase and situation model, by which one can distinguish more readily between memory of text and learning from text (Kintsch, 1994, p. 294; Kintsch, 1998, pp. 103-107). Note that these three components should not be seen as separate networks in memory, as the mental representation is considered a unitary structure. But these components can be useful when describing mental representations, and thus function as an analytical tool. It could also be noted that this tool is not directly applicable to mental representations in general, but describes mental representations in relation to a given text.

Surface component.

When the words and phrases themselves are encoded in the mental representation (possibly together with linguistic relations between them), and not the *meaning* of the words and phrases, one can talk about a surface component of the mental representation. A surface component is always present when reading, although to different degrees, since "it is generally the case, that at least some of the exact words and phrases are remembered" (Kintsch, 1998, p. 105), even when understanding the meaning of the text.

Textbase.

The textbase is a network that represents the meaning of the text, that is, the semantic structure of the text, and it "consists of those elements and relations that are directly derived from the text itself [...] without adding anything that is not explicitly specified in the text" (Kintsch, 1998, p. 103). The complete textbase would be the result of a careful and complete analysis of a text, and can be used as a reference when analyzing the mental representations of real readers, who of course do not usually create this complete textbase. Note that since the textbase consists of the *meaning* of the text, certain reformulations with respect to the text can be made, and a textbase can thus be created without any memory of the exact words or phrases from the text.

Situation model.

The pure textbase can often be "an impoverished and often even incoherent network" (Kintsch, 1998, p. 103), and to make more sense of the text, the reader must use prior knowledge to create a more complete and coherent mental representation. A construction that integrates the textbase and relevant aspects of the reader's knowledge is called the situation model. This integration can sometimes better be described as a transformation of the textbase into the situation model, since experiments have shown examples of readers who had no memory of the text itself, but who "had understood the texts very well and were able to form stable situation models on the basis of which they could answer questions correctly and make inferences about the text" (Kintsch, 1998, p. 105).

If no relevant connections are made between the text and prior knowledge, it seems not reasonable to claim that the reader has *learned* anything from the text, since the memory of the text will be reduced to the reading situation itself, and the text can only be (partially) reproduced and not used in other situations (Kintsch, 1994, p. 294). Thus, learning from a text is closely connected to the creation of a situation model, while memory of a text and in some sense understanding the text is connected to the creation of a textbase.

Mathematical Texts

In this section, some different aspects of mathematical texts will be discussed in relation to mental representations, something that will be further examined and discussed in the present study.

Surely, one can agree with Pimm (1989) and Woodrow (1982), who point out that the symbolic aspect of mathematics is one of the subject’ s most essential distinguishing features. This of course also has its effects on the reading of mathematical texts and the mental representations created when reading. For example, as Pimm discusses (1989, p. 180), in contrast to ordinary words, many mathematical symbols cannot be read unless you know the meaning of the symbol, or at least have learnt specifically how to read the symbol. So, the symbol and how it is read might very well be represented mentally quite differently when reading a mathematical text, where the symbol might be viewed more like a picture than something that can be pronounced. This can of course affect all levels of the mental representation, but to get beyond the surface component one surely needs to know how the symbols are read.

When the use of symbols becomes the only aspect of part of a text, for example when solving an equation in several steps, Woodrow (1982, p. 290) points to a phenomenon he calls temporary redundancy, “ in which a whole group of symbols are at one stage carried without reading, only to need detailed reading later.” This group of symbols are then, at least temporarily, represented mentally as one element, an action that needs an incentive, which may come directly from statements or properties in the text (e.g. by a separate heading or by indenting in the text), and thus creating a textbase; or it may in some way come from previous knowledge (e.g. about common structures in mathematical texts), and thus creating a situation model.

Two major types of mathematical texts can be those describing and explaining concepts or properties of different types, and those describing a kind of procedure or algorithm. (This distinction might also cover much of mathematics as a whole.) The latter type might often include, or totally consist of, a concrete example of how to use this procedure/algorithm, from which it would be highly essential to form a situation model, since a textbase would just be a description of this particular example. But for texts describing concepts, the textbase might be of some more use since for example some properties can be directly formulated in the text.

Purpose of This Study

This study is part of a larger project about the activity of learning mathematics by reading, where the main purpose is to contribute to the creation of a theoretical model for this activity, which in a longer perspective hopefully can be used as guidance when writing mathematical textbooks or in teaching situations where reading mathematical texts play an important role.

There seem not to exist many studies focusing on learning mathematics by reading, especially that separate *understanding a text* and *learning from a text*. This study, together with previous work (Österholm, 2003), can thus be seen as a starting point in examining the process of learning mathematics by reading mathematical texts, with special focus on aspects discussed earlier in this paper, such as different cognitive processes and levels of mental representations, aspects that previously have been applied mainly to subject areas other than mathematics. However, this study will focus on the resulting mental representations when reading, while the process of reading and how these representations are created will not be part of this study. Different kinds of situations will be observed, where different cognitive processes are active, but focus will always be on the resulting mental representations.

Thus, the focus of attention in this study is the resulting mental representations created in situations when reading mathematical texts, where different levels of mental representations will be used as an analytical tool. At the same time, the purpose is also to evaluate this tool, to note possible advantages and limitations when using it for mathematical texts.

At the university level, students seem to a larger extent expected to read mathematical texts on their own, compared to lower levels in the school system. This makes it interesting to observe students from the last year at the upper secondary school, and especially students in the Swedish natural science programme, after which many students continue their studies at university level. Thereby, another purpose of this study is simply to see examples of students' resulting mental representations when reading mathematical texts. This can be seen as a continuation of previous work (Österholm, 2003), where focus was on students' *activities* when reading and discussing mathematical texts.

Method

With respect to empirical data, this study can be divided into two parts, where also different cognitive processes are dominant.

Part 1

Here the focus is on the automatic and unconscious cognitive processes, and students were asked to work with two pamphlets, according to the following procedure, which was not given to the students beforehand:

First pamphlet: Instructed never to go back in the pamphlets whenever continued to the next page — Give background information (gender and grades in previous mathematics courses) — Carefully read a given text — Give some comments of what they thought of the text regarding difficulty and familiarity.

Second pamphlet: In writing retell, as thoroughly as possible, the text previously read — Give an appropriate heading for the text (which was absent in the given text) — Summarize the text for an imagined classmate who has not read the text.

An important detail is that the students were not informed in advance about the fact that they later were supposed to retell the text. This was done to minimize any attempts for pure memorization of the text, and so that the dominating cognitive processes should be of the automatic and unconscious type.

In this part of the study, 27 students in their third and last year of the natural science programme at the Swedish upper secondary school participated. All came from the same mathematics class, at this time they had just finished the mathematics D course[2], and some of them had had a few lessons in the mathematics E course, where they had started with a short introduction to complex numbers, including the absolute value of complex numbers.

Three different texts were used in this part, labeled text 1a, text 1b and text 2, which gave nine students per text type. All three texts were created based on texts used at the university level, and represent some different aspects of mathematics and mathematical texts. Texts 1a and 1b both present and explain a concept, the absolute value of real numbers, while text 2 describes a method for how to perform a type of procedure, a partial fraction decomposition, by performing the procedure with a concrete example[3]. The main difference between text 1a and text 1b, is in how they present the properties and statements dealing with the absolute value of real numbers, where text 1b is a little bit longer than text 1a, and uses more ordinary words, while text 1a relies more on mathematical symbols. Both texts 1a and 1b take up no more than $^1/_4$ of a page, while text 2 takes up a whole page.

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[2] Mathematics at the Swedish upper secondary school consists of courses from A to E, where mathematics A to D are mandatory within the natural science programme, while mathematics E is optional (National Agency for Education, 2001).

[3] About half of text 2 describes, step by step, how to find the constants *A* and *B* in the partial fraction decomposition: $\frac{7x-1}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$

The collected data in the form of writings from the students can of course not be said to show directly their complete mental representations of the texts, but the variety of tasks should at least show some different aspects of the students' mental representations. For example, it can be presumed that the task of retelling the text will not show much of a possible existing situation model since much focus might be on the exact content of the text. But the task of summarizing the text to an imagined classmate seems to have more potential in showing a possible created situation model. So, the students' writings as a whole will be analyzed with the help of the different components of mental representations.

Part 2

The data collected in a previous study (Österholm, 2003) will be used here, where students together in pairs read and discussed a text and solved tasks relevant to the text, which was about the absolute value of real numbers. The first part of this text is used as text 1a in part 1 of the present study. After the students' activities, a short interview was also carried out, with questions about their discussions of the text and tasks.

Four students in their third and last year of the natural science programme at the Swedish upper secondary school participated. All came from the same mathematics class, and at this time they were taking the mathematics D course. Their discussions together with the following interview were videotaped and transcribed, which together with students' notes from solving the tasks are the material used in the present study. In contrast to part 1, the students' cognitive processes in this part can essentially be characterized as problem solving, when discussing the text and the tasks.

The data used in this part give a more indirect picture of the students' mental representations than is the case in part 1, since the students are not directly asked to formulate their memory for, or understanding of, the text. But with the help of the quite long observations of the students' discussions about the text and the tasks it should be possible to infer a picture of the students' mental representations.

Results

In this section, the results from the two parts of this study will be treated separately, but a comparison of these results together with the analysis using the different levels of mental representations will be done in the next section; Discussion and Conclusions.

Part 1

Students with different grades in mathematics were quiet evenly distributed in the three groups who read the three different texts. The male-female ratio was as close to 0.5 as possible in all groups. The groups are thereby in principle comparable with respect to grade and gender, but it should be remembered that there are rather few participating students, and there will not be any far-reaching comparisons.

For all three texts, the students' writings were a bit more concise and fragmental than was anticipated. The results may thereby say more about (Swedish) students' reading of mathematical texts than about any general aspects of reading mathematical texts. But the students' writings are still a result from reading the texts, and therefore it is believed that this part will still function as an important test of the methods and analyzing tools used in this study. Another general result for all three texts is that there is a clear tendency for students with higher grades to recall more correct statements from the text than students with lower grades. Some type of prior knowledge thus seems to come into play when reading these texts. The continued analysis in this part is built up around the comparisons between the different texts.

Text 1a compared to text 1b.

The number of students answering that the text was easy/hard to read was $^4/_5$ for those reading text 1a and $^6/_3$ for those reading text 1b. Regarding whether the content of the text was familiar, the number of students answering yes/partly/no was 1/2/6 for those reading text 1a and 5/3/1 for those reading text 1b. Thus, these students seem to prefer text 1b, which perhaps is not much of a surprise since this text uses words and formulations that are more common, and not only special mathematics words and symbols. However, when the students get to retell the text, there is no significant difference between the two groups regarding how much is recalled or what parts of the text are recalled, where the focus among all students is on retelling the symbols from the text, and especially the definition of absolute value[4]. Almost half of all students who read either text 1a or text 1b do not use a word for absolute value, neither in their recalls nor in the explanations to the imagined classmate, but only use the symbol. However, when asked to summarize the text for an imagined classmate, a difference between the two groups can be noticed. The summaries given by students who have read text 1a are quite similar to what was written when recalling the text, that is, focus is still on symbols, while the students who have read text 1b give a summary using fewer symbols than when retelling the text. However, all students have difficulties in formulating the explanations, and it is common that different parts of the text are mixed, as the students seldom reformulate, but reproduce parts of the text, also when giving a summary. Thus, both when retelling and when giving a summary, the students are trying to reproduce the text.

When retelling symbols, the students' writings are sometimes fragments of a statement, or rather strange combinations of different symbols, and the same can happen when students write sentences where ordinary words are mixed with symbols, but this sort of fragmentation never happens using only ordinary words. Thus, even if a complete ordinary sentence is not remembered by the students, they seem to *construct* a grammatically correct statement from the parts remembered, which is not the case when using mathematical symbols.

Texts 1a and 1b compared to text 2.

There is a big difference between what the students recall of text 2 compared to what they recall of texts 1a and 1b. Although there are quite a lot of symbols also in text 2, almost all students write no symbols at all when retelling text 2. But it could be noted that the symbols have quite different roles in these texts, where in text 2 there are algebraic expressions, equations, and manipulations of these, something that most likely is highly familiar to the students. While in texts 1a and 1b, the use and meaning of the (for some students; new) symbol of absolute value is explained. That is, in text 2 the focus is on the procedure as a whole where symbols together with explanations in ordinary words build up this procedure, while in the texts about absolute value, the symbols themselves are the focus of attention. The fact that text 2 is longer than the other texts can of course also affect the type of recall given, since it surely is harder to remember details when reading a longer text. These two aspects might contribute to the observed differences when retelling the texts.

When the students retell text 2, they seldom give any direct quotations from the text, but often formulate *description* of different parts of the text, which seldom are related to each other. For example, some students write that two constants A and B were to be calculated, but do not refer to where these constants come from. Another common comment is that there was a system of equations at the end of the text, often without any connection to what these equations solved. It thus seems like there are individual words or parts of the text that are remembered, but the

[4] The definition, as it is used in both texts, consists of:
$| x | = x$ if $x \geq 0$
$| x | = - x$ if $x \leq 0$

relations between these parts are not often remembered or understood. This can result in a rather fragmented retelling of the text that do not capture the essence of the whole text, something that perhaps is not to expect when asking the students to *retell* the text, and the students' answers to the task to explain the text to an imagined classmate show another side of the their mental representations. Unlike the results from texts 1a and 1b, the writings on this task for text 2 are for most students much different from when retelling the text. Here, the students freely formulate their views of what the whole text is about, and parts of the text are no longer reproduced, as was the most common case for texts 1a and 1b. The results on this task show that most of the students have a common view of what the text is about, where many students use statements similar to 'split fractions into two'. However, no student explains the procedure used for how to do this 'splitting of fractions'.

Part 2

As already mentioned, this part uses empirical data collected in a previous study (Österholm, 2003), and a more detailed discussion about the results can of course be found in that study. As was then noted, the participating two pairs of students were not comparable with respect to prior knowledge about absolute value, but the only purpose of comparing them was to explore differences in their discussions, and in the present study the purpose is to explore differences in their resulting mental representations after reading and discussing the text. It was also noted that the two pairs of students acted very differently, and their mental representations of the text are very different.

After concluding, with direct reference to the heading, that the text is about absolute value, the first[5] pair cannot manage to give a short summary of the text and they have great difficulties in explaining the meaning of absolute value. In their discussions, the students in this pair also did not use the term *absolute value*, or any other word, for the symbol of absolute value, and they are clearly puzzled by the 'vertical lines' in the symbol. The students in the second pair do use the term absolute value when referring to the symbol, and they also 'create' words for other properties of the absolute value, for example a word referring to the value of x where, according to the definition, an expression of the type $| x - 3 |$ should be split into two cases, in this example; when $x = 3$.

The students in the first pair seem to have rather fragmented mental representations of the text, since they can refer to different parts of the text, but never seem to relate or combine these parts. This becomes evident when asked to explain the meaning of absolute value, and they give a description of a few parts of the text, referring to the given example of solving an equation and to the definition, but without giving any details, and without relating these two parts of the text. The second pair had previously had some experience of the absolute value from the function on their calculators, and their prior knowledge consisted of the view that the absolute value 'makes negative numbers positive'. The different parts of the text are often related to or interpreted with the help of this prior knowledge, which seems to make their mental representations more structured and coherent, where different parts of the text are often related to each other.

Finally, it can be noted that the students in the second pair at a few times try to explore some different generalizations of different aspects of the given example of solving an equation. Thus, they seem to create a mental representation of the given example in a more generalized form than is presented in the text, while the students in the first pair focus on the specific example, and seem to remember more details from the example.

[5] As labeled in the previous study (Österholm, 2003).

Discussion and Conclusions

Different kinds of cognitive processes come into play in the two parts of this study, and the first part shows a general need for the students to reflect more on the content of the text for better understanding. However, the results from the second part show that the opportunity to discuss and reflect upon the text not automatically renders a much deeper understanding of the content, but the second part also gives examples of the potential for creating more elaborate mental representations in a more problem-solving activity when reading mathematical texts. Many students seem to have problems to *decode* the text, that is, they have problems grasping the structure and meaning of each separate sentence and statement. Thus, the students have problems to create a textbase in the mental representation of the text, and the surface component often seems to dominate in the students' mental representations, where individual words or symbols are often remembered, but not as often any complete statements, something that becomes most evident in parts of the texts where symbols are used. At the same time, as is shown in both parts, the students focus on the symbolic expressions in the text, a focus that perhaps is influenced by their general views about mathematics, where one student even writes that he at first did not realize that he "was allowed to use ordinary words" when retelling the text. Due to the problems to decode the statements in the text, the focus on symbols tends to be limited to the symbols themselves and not the meaning of the symbols. Pimm (1989, p. 157) discusses this phenomenon as a more general situation in mathematics, where "the pupil's attention [is kept] on the mastery of the production of the symbols themselves, rather than on trying to grasp or express the meaning which they represent." Some of the students participating in this study also seem to regard the use of symbols in the text as a problem. For example, one student expresses the wish for the text to use ordinary words instead of symbols, giving an example that it would be better to write 'for numbers less than or equal to zero' instead of using the expression ' $x \leq 0$' . Thus, the students' mental representations of symbols sometimes seem to have only weak connections to the mental representation of the corresponding word (i.e. how the symbol is read). Of course, this makes it more difficult to focus on the meaning of the text since the symbols more or less are seen as pictures, and the mental representation is thereby likely to be limited to the surface component. This relationship was discussed earlier (pp. 4-5), and the results show that this is an important aspect when reading mathematical texts.

In most situations observed in this study, the students seem to *reproduce* the text content, where individual words and symbols are often remembered together with some complete statements (not necessarily remembered verbatim). Not often does a *reconstruction* of the text seem to occur, where the content of the text would be rebuilt from the memory of what the text was about, with a focus on the *meaning* of the text, and not on specific statements and formulations, but on the text as a whole. Thus, the students' mental representations seem to a great deal consist of a surface component and/or a textbase, from where a text reproduction is possible, but the situation models created by the students, from where it could be possible to reconstruct a text, seem not to be very elaborate. To be able to create a good situation model of the text, appropriate prior knowledge is needed. Different kinds of prior knowledge can affect the situation model (and the mental representation in general), such as knowledge of words and concepts used in the text, and knowledge of the use of variables in mathematics. The concept 'real numbers' is used in the texts about absolute value, and several students evidently have a very limited knowledge of this concept, where some students in part 1 come to believe that the text is introducing and explaining the concept of real numbers. Problems with the use of variables were also observed, where for example one student writes "for negative x, that is $-x$", as if variables in themselves cannot be negative. These are examples of how certain limits in prior knowledge can affect the students' understanding of the text. The second pair in part 2 gives the most evident example of how prior knowledge can help in the creation of a situation model, where different parts of the text are interpreted or related to each other with the help of their prior knowledge of absolute value as ‘making the numbers positive’ . These students also build up the situation model in other ways, for example by discussing generalizations of statements and examples given in the text, and thereby not primarily remembering the exact formulations from the text.

Prior personal experiences can of course also affect the creation of mental representations, and the students’ prior usage of mathematical texts seems to influence them when reading. A student from part 2 describes the usage of the textbook as “first there are explanations and derivations, then examples, and then similar tasks, and then you can always look back at the examples and see how they solved them, and from that solve the tasks”, and both pairs focus to a great deal on the given example in the text. Thus, the students’ prior experiences with mathematical texts and mathematics in general, seem to cause them to focus, consciously and/or unconsciously, on certain parts of the mathematical texts, for example symbolic expressions and given examples, as has been observed in this study.

In text 2, almost no student remembers any of the algebraic expressions in the example of partial fraction decomposition, something that perhaps could be explained with the so-called temporary redundancy (see page 5). However, the students’ mental representations of these expressions seem to very simplified, in the sense that the expressions seem to be represented only as ‘an expression’, where no properties of this expression are preserved. There is no need for any particular prior knowledge for this representation, and it gives a surface component or perhaps a textbase in the mental representation. If an expression of the type $x^2 + 3x + 7$ should be represented as ‘a polynomial of degree two’, a situation model would be created (provided that the text is not referring to this expression as a polynomial of degree two) since certain prior knowledge about what is expressed in the text has been used, but something similar to this has not been observed in this study. However, when a specific expression is given in the text, the students might be unsure about using words to describe what they remember (recall their sometimes extreme focus on symbols), even if they remember what type of expression it was. Although some students did exactly this, by stating that a system of equations was used in the text, without remembering the details of the system, but the text also refers to it as a system of equations, and it is thus difficult to decide whether their memories were from the text referring the system, or the system itself. Since no student has described the general structure or method of partial fraction decomposition, no clear situation models seem to have been created from text 2. Much of the students’ mental representations could be labeled as surface component, when just remembering individual words, or as textbase, when for example stating what is meant by partial fraction decomposition from the corresponding statement in the text, as a way to split certain fractions.

Methodological Issues

As has been discussed, students’ prior experiences in mathematics and their views of the subject influence their understanding of mathematical texts. However, students’ experiences and views, for example about what one can and cannot do or write, could also affect how they respond to given tasks, so that the methods used in this study do not give an as accurate picture of their mental representations as possible. Also, if the students have never read mathematical texts for the purpose of learning something or discussing the content afterwards, there is a risk that they did not read the text in this study as thoroughly as possible either.

Regarding the first part of this study, one can perhaps question the possibility of grasping a text just by reading it once, and the heavy use of symbols might make this even more difficult. There might also be another special aspect of mathematics that makes the reading more difficult, in that the focus in mathematics often is the relationships between statements (if-then relationships), something that can make the structure of a mathematical text rather complex. All together, there might be a general need to read a mathematical text more than once to create more elaborate mental representations.

Conclusions

In thus study, certain aspects of students' reading and understanding of mathematical texts have been noticed, aspects that are closely related to special properties of mathematics and students' experiences of the subject. The symbolic language is a very important part of mathematics, but also gives rise to some potential problems when reading mathematical texts, for example regarding the relation between the symbol and how it is read, where students often seem to view the symbols as figures that are not articulated when reading the text. At the same time, the students seem to have more focus on symbolic parts of texts than other parts that use a more ordinary language. Many students also seem to have problems creating more than a surface component or a textbase when reading mathematical texts, something that also seems closely related to the use of mathematical symbols. With such limited situation models that have been observed for many students, it becomes difficult to claim that they have *learned* very much by reading these texts.

The notion of different levels of mental representations of texts has proven to be a useful tool when analyzing, describing and characterizing students' memory and understanding of mathematical texts, and it has helped to highlight certain interesting aspects special to reading *mathematical* texts.

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